Hypothesis Testing Details: For $\mu_1 - \mu_2$, large samples

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad \mu_1 - \mu_2 \leq \Delta_0$ $\quad \mu_1 - \mu_2 \geq \Delta_0$
   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad \mu_1 - \mu_2 > \Delta_0$ $\quad \mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean __________ for all _______________
   and μ_2 is the mean __________ for all _______________

2. **Assumptions:** We have independent, random samples from two populations, and the sample size from each is large enough that we can use the Central Limit Theorem.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < Z_\alpha$
   Right: Reject H_0 if $TS > Z_{1-\alpha}$
   Two: Reject H_0 if $\mid TS \mid > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5. **P-value:** For the three types of tests:

   Left: $P - \text{value} = P(Z < TS)$
   Right: $P - \text{value} = P(Z > TS)$
   Two: $P - \text{value} = 2 \cdot P(Z < - \mid TS \mid)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean __________ for all _______________ and the mean __________ for all _______________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For $\mu_1 - \mu_2$, Small sample and σ_1, σ_2 unknown but equal, normal data

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad\mu_1 - \mu_2 \leq \Delta_0$ $\quad\mu_1 - \mu_2 \geq \Delta_0$

   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad\mu_1 - \mu_2 > \Delta_0$ $\quad\mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean ____________ for all __________________
   and μ_2 is the mean ____________ for all __________________

2. **Assumptions:** We have independent, random samples from two normally distributed populations, with variances that are unknown but equal.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < t_{n_1+n_2-2,\alpha}$

   Right: Reject H_0 if $TS > t_{n_1+n_2-2,1-\alpha}$

   Two: Reject H_0 if $\mid TS \mid > t_{n_1+n_2-2,1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{Where: } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

5. **P-value:** For the three types of tests:

   Left: $P - \text{value} = P(t_{n_1+n_2-2} < TS)$

   Right: $P - \text{value} = P(t_{n_1+n_2-2} > TS)$

   Two: $P - \text{value} = 2 \cdot P(t_{n_1+n_2-2} > \mid TS \mid)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean ____________ for all __________________ and the mean ____________ for all __________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For $\mu_1 - \mu_2$, Small sample and σ_1 and σ_2 unknown, normal data

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad$ $\mu_1 - \mu_2 \leq \Delta_0$ $\quad$ $\mu_1 - \mu_2 \geq \Delta_0$
   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad$ $\mu_1 - \mu_2 > \Delta_0$ $\quad$ $\mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean ____________ for all __________________
   and μ_2 is the mean ____________ for all __________________

2. **Assumptions:** We have independent, random samples from two normally distributed populations, with variances that are unknown.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < t_{df,\alpha}$
   Right: Reject H_0 if $TS > t_{df,1-\alpha}$
   Two: Reject H_0 if $| TS |> t_{df,1-\alpha/2}$

   $$\text{Where: } df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$$

4. **Test Statistic:**

   $$TS = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5. P**-value:** For the three types of tests:

   Left: $P - \text{value} = P(t_{df} < TS)$
   Right: $P - \text{value} = P(t_{df} > TS)$
   Two: $P - \text{value} = 2 \cdot P(t_{df} >| TS |)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean ____________ for all __________________ and the mean ____________ for all __________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For σ_1^2/σ_2^2, normal data

1. **Hypotheses:**

   H_0: $\frac{\sigma_1^2}{\sigma_2^2} = \Lambda_0$ $\quad \frac{\sigma_1^2}{\sigma_2^2} \leq \Lambda_0$ $\quad \frac{\sigma_1^2}{\sigma_2^2} \geq \Lambda_0$

   H_0: $\frac{\sigma_1^2}{\sigma_2^2} \neq \Lambda_0$ $\quad \frac{\sigma_1^2}{\sigma_2^2} > \Lambda_0$ $\quad \frac{\sigma_1^2}{\sigma_2^2} < \Lambda_0$

   Where: σ_1^2 is the variance associated with the measurement of ______________ for all ____________________
   and σ_2^2 is the variance associated with the measurement of ______________ for all ____________________

2. **Assumptions:** We have independent random samples from two normally distributed populations.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < F_{n_1-1,n_2-1,\alpha}$
   Right: Reject H_0 if $TS > F_{n_1-1,n_2-1,1-\alpha}$
   Two: Reject H_0 if $TS < F_{n_1-1,n_2-1,\alpha/2}$ or if $TS > F_{n_1-1,n_2-1,1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{s_1^2}{s_2^2 \cdot \Lambda_0}$$

5. ***P*-value:** For the three types of tests:

   Left: $P - \text{value} = P(F_{n_1-1,n_2-1} < TS)$
   Right: $P - \text{value} = P(F_{n_1-1,n_2-1} > TS)$
   Two: $P - \text{value} = 2 \cdot P(F_{n_1-1,n_2-1} < TS)$ or $P - \text{value} = 2 \cdot P(F_{n_1-1,n_2-1} > TS)$, whichever is less than 1.

6. **Conclusion:** There is enough evidence to conclude that ratio of the variance associated with the measurement of ______________ for all ____________________ and the variance associated with the measurement of ______________ for all ____________________ (is more than/less than/not) (value of Λ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For $\mu_1 - \mu_2$, Small sample, non-normal data, n_1, n_2 greater than 8

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad$ $\mu_1 - \mu_2 \leq \Delta_0$ $\quad$ $\mu_1 - \mu_2 \geq \Delta_0$

   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad$ $\mu_1 - \mu_2 > \Delta_0$ $\quad$ $\mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean ____________ for all ____________________
   and μ_2 is the mean ____________ for all ____________________

2. **Assumptions:** We have independent random samples from two populations that have distributions with the same shape.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < Z_{\alpha}$

   Right: Reject H_0 if $TS > Z_{1-\alpha}$

   Two: Reject H_0 if $\mid TS \mid > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{W - \mu_w}{\sigma_w}$$

Where: R_j is the rank of $(x_{1j} - \Delta_0)$ in the combined sample; $W = \sum_{j=1}^{n_1} R_j$

$$\mu_w = \frac{n_1(n_1 + n_2 + 1)}{2} \qquad \sigma_w^2 = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} - \frac{n_1 n_2 \sum(\tau_j - 1)(\tau_j)(\tau_j + 1)}{12(n_1 + n_2)(n_1 + n_2 - 1)}$$

and τ_j is the frequency of the j^{th} distinct value in the combined sample

5. **P-value:** For the three types of tests:

   Left: $P-\text{value} = P(Z < TS)$

   Right: $P-\text{value} = P(Z > TS)$

   Two: $P-\text{value} = 2 \cdot P(Z > \mid TS \mid)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean ____________ for all ____________________ and the mean ____________ for all ____________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For $\mu_1 - \mu_2$, Paired data, small sample

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad$ $\mu_1 - \mu_2 \leq \Delta_0$ $\quad$ $\mu_1 - \mu_2 \geq \Delta_0$
   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad$ $\mu_1 - \mu_2 > \Delta_0$ $\quad$ $\mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean ____________ for all __________________
   and μ_2 is the mean ____________ for all __________________

2. **Assumptions:** We have a random sample from a normally distributed population of differences, with unknown variance.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < t_{n-1,\alpha}$
   Right: Reject H_0 if $TS > t_{n-1,1-\alpha}$
   Two: Reject H_0 if $\mid TS \mid > t_{n-1,1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\bar{D} - \Delta_0}{\frac{s}{\sqrt{n}}}$$

5. P**-value:** For the three types of tests:

   Left: $P - \text{value} = P(t_{n-1} < TS)$
   Right: $P - \text{value} = P(t_{n-1} > TS)$
   Two: $P - \text{value} = 2 \cdot P(t_{n-1} > \mid TS \mid)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean ____________ for all __________________ and the mean ____________ for all __________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For $\mu_1 - \mu_2$, Paired data, large sample

1. **Hypotheses:**

   H_0: $\mu_1 - \mu_2 = \Delta_0$ $\quad \mu_1 - \mu_2 \leq \Delta_0$ $\quad \mu_1 - \mu_2 \geq \Delta_0$
   H_a: $\mu_1 - \mu_2 \neq \Delta_0$ $\quad \mu_1 - \mu_2 > \Delta_0$ $\quad \mu_1 - \mu_2 < \Delta_0$

   Where: μ_1 is the mean ____________ for all __________________
   and μ_2 is the mean ____________ for all __________________

2. **Assumptions:** We have a random sample from some population of differences, and the sample size is large enough that we can use the Central Limit Theorem.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < Z_\alpha$
   Right: Reject H_0 if $TS > Z_{1-\alpha}$
   Two: Reject H_0 if $\mid TS \mid > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\bar{D} - \Delta_0}{\frac{\sigma}{\sqrt{n}}}$$

5. ***P*-value:** For the three types of tests:

   Left: $P - \text{value} = P(Z < TS)$
   Right: $P - \text{value} = P(Z > TS)$
   Two: $P - \text{value} = 2 \cdot P(Z > \mid TS \mid)$

6. **Conclusion:** There is enough evidence to conclude that difference between the mean ____________ for all __________________ and the mean ____________ for all __________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: “In fact, it is (more/less).”)

Hypothesis Testing Details: For $p_1 - p_2$

1. **Hypotheses:**

   H_0: $p_1 - p_2 = \Delta_0$ $\quad p_1 - p_2 \leq \Delta_0 \quad p_1 - p_2 \geq \Delta_0$

   H_a: $p_1 - p_2 \neq \Delta_0$ $\quad p_1 - p_2 > \Delta_0 \quad p_1 - p_2 < \Delta_0$

   Where: p_1 is the true proportion of all ____________ that __________________ and p_2 is the true proportion of all ____________ that __________________

2. **Assumptions:** We have independent, random observations from two binomial experiment, and there are enough trials in each that we can use the Central Limit Theorem.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < Z_\alpha$

   Right: Reject H_0 if $TS > Z_{1-\alpha}$

   Two: Reject H_0 if $| TS | > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{(\hat{p}_1 - \hat{p}_2) - \Delta_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

5. ***P*-value:** For the three types of tests:

   Left: $P - \text{value} = P(Z < TS)$

   Right: $P - \text{value} = P(Z > TS)$

   Two: $P - \text{value} = 2 \cdot P(Z > | TS |)$

6. **Conclusion:** There is enough evidence to conclude that difference between the proportion of all ____________ that __________________ and the proportion of all ____________ that __________________ (is more than/less than/not) (value of Δ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")